Inference at $*$ 1 0
of proof for Lemma order_functionality_wrt_iff:

1. T : Type
2. R : $T \rightarrow T \rightarrow \mathbb{P}$
3. R' : $T \rightarrow T \rightarrow \mathbb{P}$
4. $\forall x, y{:}T.\ R(x,y) \iff R'(x,y)$

$\vdash$ (Refl(T;x,y.R(x,y)) & Trans(T;x,y.R(x,y)) & AntiSym(T;x,y.R(x,y)))
$\iff$ (Refl(T;x,y.R'(x,y)) & Trans(T;x,y.R'(x,y)) & AntiSym(T;x,y.R'(x,y)))
by PERMUTE{1:n,
2:n,
2:n,
2:n,
3:n,
4:n,
5:n,
6:n,
7:n,
8:n,
9:n,
10:n,
11:n,
12:n,
7:n,
13:n,
14:n,
15:n,
16:n,
7:n,
17:n,
18:n,
19:n,
20:n,
21:n,
7:n,
7:n,
8:n,
9:n,
10:n,
11:n,
12:n,
7:n,
22:n,
23:n}

1: . . . . .wf. . . . . NILNIL

$\vdash$ (Refl(T;x,y.$R(x,y)$) & Trans(T;x,y.$R(x,y)$) & AntiSym(T;x,y.$R(x,y)$)) $\in \mathbb{P}$

2: . . . . .wf. . . . . NILNIL

$\vdash$ (Refl(T;x,y.$R'(x,y)$) & Trans(T;x,y.$R'(x,y)$) & AntiSym(T;x,y.$R'(x,y)$)) $\in \mathbb{P}$

3: . . . . .wf. . . . . NILNIL

$\vdash$ Refl(T;x,y.$R(x,y)$) $\in \mathbb{P}$

4: . . . . .wf. . . . . NILNIL

$\vdash$ Refl(T;x,y.$R'(x,y)$) $\in \mathbb{P}$

5: . . . . .wf. . . . . NILNIL

$\vdash$ (Trans(T;x,y.$R(x,y)$) & AntiSym(T;x,y.$R(x,y)$)) $\in \mathbb{P}$

6: . . . . .wf. . . . . NILNIL

$\vdash$ (Trans(T;x,y.$R'(x,y)$) & AntiSym(T;x,y.$R'(x,y)$)) $\in \mathbb{P}$

7: . . . . .wf. . . . . NILNIL

$\vdash T \in$ Type

8: . . . . .wf. . . . . NILNIL

$\vdash (\lambda x,y.\ R(x,y)) \in T \rightarrow T \rightarrow \mathbb{P}$

9: . . . . .wf. . . . . NILNIL

$\vdash (\lambda x,y.\ R'(x,y)) \in T \rightarrow T \rightarrow \mathbb{P}$

10: . . . . .wf. . . . . NILNIL

5. $x : T$
6. T
$\vdash x \in T$

11: . . . . .wf. . . . . NILNIL

5. T
6. $y : T$
$\vdash y \in T$

12: . . . . .wf. . . . . NILNIL

5. T
$\vdash T \in$ Type

13: . . . . .wf. . . . . NILNIL

$\vdash$ Trans(T;x,y.$R(x,y)$) $\in \mathbb{P}$

14: . . . . .wf. . . . . NILNIL

$\vdash$ Trans(T;x,y.R'(x,y)) $\in \mathbb{P}$
15: .....wf..... NILNIL

$\vdash$ AntiSym(T;x,y.R(x,y)) $\in \mathbb{P}$
16: .....wf..... NILNIL

$\vdash$ AntiSym(T;x,y.R'(x,y)) $\in \mathbb{P}$
17: .....wf..... NILNIL

$\vdash$ (λy,x. R(x,y)) $\in T \rightarrow T \rightarrow \mathbb{P}$
18: .....wf..... NILNIL

$\vdash$ (λy,x. R'(x,y)) $\in T \rightarrow T \rightarrow \mathbb{P}$
19: .....wf..... NILNIL

5. T
6. $x : T$
$\vdash x \in T$
20: .....wf..... NILNIL

5. $y : T$
6. T
$\vdash y \in T$
21: .....wf..... NILNIL

5. T
$\vdash T \in$ Type
22: .....wf..... NILNIL

$\vdash$ (Refl(T;x,y.R'(x,y)) & Trans(T;x,y.R'(x,y)) & AntiSym(T;x,y.R'(x,y)))
=
(Refl(T;x,y.R'(x,y)) & Trans(T;x,y.R'(x,y)) & AntiSym(T;x,y.R'(x,y)))
23:

$\vdash$ (Refl(T;x,y.R'(x,y)) & Trans(T;x,y.R'(x,y)) & AntiSym(T;x,y.R'(x,y)))
$\iff$ (Refl(T;x,y.R'(x,y)) & Trans(T;x,y.R'(x,y)) & AntiSym(T;x,y.R'(x,y)))
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